run on the IBM 7094, model I, and the results are summarized in Table 1.

Furthermore, in many problems, the total time for both relabeling and solution of the equations, where two or more unknowns were associated with each node, was less than the solution time without relabeling.

As a concluding remark, we have to note that single interchanges with repeated sweeping procedures are very promising. It is always possible to improve the existing provisions of the programs or implement various "look ahead" features without altering the basic philosophy of the scheme.

Errata†

The following errors and basic differences from the working program have been discovered in Ref. 3 in the listing of SUBROUTINE ARAN: 1) statement below 1105 should read IF (XSA-XSAP) 1107, 1107, 1108; 2) statement above 1290 should read IF (IJ-IN) 1290, 1290, 1310; 3) statement above 1600 should read CALL SEBIN (BCH, JBIP, NCH). In addition to these, the authors suggest that the empirical constant NCYCN = 3 + IN/100 be changed to NCYCN = IN to improve the chances of obtaining optimum bandwidth at the expense of increasing computation time.

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† The author wishes to thank A. Whitney of McDonnell Douglas Corp. for pointing out the errors.

The Siren Revisited

WILLIAM P. RODDEN*
Los Angeles, Calif.

IN a recent Note, Myklestad[1] recalls the glory that was once the desk calculator. He compares matrices to the Sirens. "Your next land-fall will be upon the Sirens: and these craze the wits of every mortal who gets so far. If a man come on them unwittingly and lend ear to their Siren-voices, he will never again behold wife and little ones rising to greet him with bright faces when he comes home from the sea." (Circe's warning to Odysseus, Ref. 2, p. 170.) This writer prefers to regard matrices as providing the means of leading Odysseus (the structural dynamicist and aeroelastician) past the Sirens (the Myklestad vibration analysis[3]?), between Scylla (the computer systems programmer) and Charybdis (the computer), and safely home to Penelope (the optimum structure).

The idealization of current aerospace structural designs as statically determinate systems of bending and twisting beam elements located along an elastic (elusive?) axis is an anachronism. The approximation may still be adequate for certain preliminary design purposes, but it can hardly be regarded as a reliable means to arrive at an optimum design for minimum weight with sufficient strength, stiffness, and life. Matrices of structural[4] and aerodynamic[5] influence coefficients do provide such a means. Beam methods may still continue to be useful for analysis of helicopter blades and long, slender missiles, but their utility in optimizing the design of highly swept or low-aspect ratio surfaces and large-diameter shell missile or fuselage structures for high-performance aerospace vehicles is gone with the wind—like the City of Troy.

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* Consulting Engineer. Associate Fellow AIAA.

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Reply by Author to W. P. Rodden

N. O. MYKLESTAD*
The University of Texas at Arlington, Arlington, Texas

MR. Rodden seems to have missed the point of my Note, since I was not comparing different methods of analyses at all. The gist of my message was that any method of analysis could be arranged in various ways for efficient numerical computations, and, if matrices were used for this purpose, it could result in greatly increased machine time. The real advantage of using matrices is in the simplication of the program, but, if the calculations are to be performed frequently, it may pay to write the program without the use of matrices.

Of course, if a matrix method of analysis is used the programming is automatically done in matrix form; but the method referred to in my Note was developed without the use of matrices, and later matrices were introduced ostensibly to make the method more efficient. However, the introduction of matrices in this case reduced the efficiency of the computer program by more than 50%.

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* Professor of Engineering Mechanics.

Comments on "Natural Frequencies of Clamped Cylindrical Shells"

GEOFFREY B. WARBURTON,*
University of Nottingham,
Nottingham, United Kingdom

IN a recent paper, Smith and Haft[1] used Flügge's equations,[2] as uncoupled by Yu,[3] to determine natural frequencies of clamped cylindrical shells. Nondimensional

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* Professor of Applied Mechanics.

Table 1 Frequencies (cps), and percentage differences in frequency, for a steel, clamped-clamped cylindrical shell[a]

m/n		2	3	4	5	6
1	A^b	1240	2150	3970	6320	9230
	B^b	1288	2109	3886	6240	9131
	C^b	2.67	0.38	0.12	0.09	0.07
2	A	2440	2560	4160	6475	9380
	B	2592	2550	4079	6382	9259
	C	6.74	1.71	0.39	0.21	0.17
3	A	...	3380	4540	6720	9540
	B	4249	3384	4467	6641	9480
	C	6.34	2.89	0.78	0.32	0.22
4	A	....	4480	5130	7100	9890
	B	5997	4489	5070	7035	9805
	C	5.05	3.24	1.18	0.46	0.27

[a] Length l = 15.65 in., mean radius a = 1.924 in., thickness h = 0.101 in.
[b] A = experimental frequency[5]; B = exact frequency; C = percentage difference between approximate and exact frequencies.

frequency factors for clamped cylindrical shells for a wide range of parameters, derived from Flügge's equations in a manner similar to that of Smith and Haft but not using the uncoupling conditions of Yu, have been published recently.[4] As there are significant differences in natural frequencies obtained from the two theories, some detailed comparison is useful.

In Table 1 the frequencies of a steel circular cylindrical shell for various values of the circumferential wave number n and the number of axial half waves m for $n \geqslant 2$ are given; the exact values have been computed from the complete solution of Flügge's equations,[4] and are compared with experimental results.[5] Frequencies for a cylindrical shell, having the same dimensions and elastic properties, were computed by Smith and Haft and given in their Table 2.[1] Comparison of the two tables shows that the frequencies in Table 1 are consistently, and sometimes considerably, lower than the values of Smith and Haft and that the agreement with experimental values is better in Table 1. Except for $n = 2$, there is a tendency for the experimental frequencies to be slightly higher than the theoretical values; thus it is not likely that imperfect clamping in the experimental setup is a cause of differences between theory and experiment.

These differences in natural frequency from the two theories can arise only from the use of Yu's uncoupling procedure by Smith and Haft. From either theory, the assumption that the variation of each component of displacement with the axial coordinate x is of the form $\exp(\lambda_i x/a)$ leads to a fourth-order equation in λ_i^2, which can be written

$$D_0\lambda_i^8 + D_1\lambda_i^6 + D_2\lambda_i^4 + D_3\lambda_i^2 + D_4 = 0 \quad (1)$$

The uncoupling procedure affects the terms occurring in the coefficients D_k; many of the differences concern small terms and have little numerical significance. However, in the coefficient D_4 there is a term $\frac{1}{2}(1 - \nu)\beta n^8$ if the uncoupling procedure is followed and a term $\frac{1}{2}(1 - \nu)\beta n^4(n^2 - 1)^2$ if direct solution of Flügge's equations is followed. The parameter $\beta = h^2/12a^2$, where h = thickness and a = mean radius. In each case this is the only term in D_4 which is independent of the frequency. The differences between these terms could account for most of the differences in natural frequency obtained by the two methods.

The effect of uncoupling can be illustrated in an alternative manner. For a cylindrical shell with simply supported ends without axial constraint, the radial component of displacement

$$w = W \cos n\phi \sin(m\pi x/l) \sin\omega t \quad (2)$$

with the integers m and n defining the mode. Similar expressions for the axial and tangential components can be written down. Substitution into any appropriate set of equations, in terms of displacements and their derivatives, for a cylindrical shell element leads to the determination of natural frequencies. In Table 2, frequencies are given for a few modes of the cylindrical shell, having the same dimensions and elastic properties as that considered in Table 1 but with simply supported ends, from Flügge's equations, from Timoshenko's equations,[6] and from Timoshenko's equations using Yu's uncoupling procedure.[3] It will be noted that the change from Flügge's to Timoshenko's equations has a very small effect on frequency and that the uncoupling procedure has an appreciable effect on frequency. Results for modes not included in Table 2 confirm these points. It seems reasonable to assume that the effect of uncoupling Flügge's equations would have a comparable effect on frequency. The effect of uncoupling on frequency for this simply supported shell is comparable to the difference in frequency between Smith and Haft's results and those in Table 1 for the clamped shell of the same dimensions.

Table 2 Frequencies for a steel, simply supported cylindrical shell[a]

m/n		2	4	6
1	A^b	905	3869	9126
	B^b	903	3867	9124
	C^b	1108	4126	9388
3	A	3837	4334	9443
	B	3831	4327	9438
	C	3909	4590	9709

[a] l = 15.65 in., a = 1.924 in., h = 0.101 in.
[b] A = frequency from Flügge's equations; B = frequency from Timoshenko's equations; C = frequency from Timoshenko's equations, uncoupled by Yu.

Table 1 gives also the percentage differences in natural frequency from the exact theory discussed here and from an approximate theory, which uses the Rayleigh-Ritz method with beam functions assumed for the variation of displacement in the axial direction. This approximate theory has been given elsewhere,[5] but in computing results from it the strain energy expressions have, for consistency, been based on Flügge's equations and not on Timoshenko's equations as they were in the original reference, although the effect on natural frequency of using these two different sets of equations is very small. It will be observed that the approximate method overestimates the natural frequencies slightly, as expected, but the percentage difference decreases as n increases. This approximate theory gives much closer agreement with the exact theory than results based on Donnell's theory, which were given by Smith and Haft in their Table 2; this is particularly true for low values of n. However, the approximate method used here involves more complex expressions than that based on Donnell's theory.

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